A Library of ADMM for Sparse and Low-rank Optimization

version 1.1

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https://github.com/canyilu/LibADMM

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1 Introduction

The LibADMM toolbox solves many popular compressive sensing problems (see Table 1) by M-ADMM proposed in [14]. Some more details will come soon.

Citing. In citing this toolbox in your papers, please use the following references [10] [14]:

Canyi Lu. A Library of ADMM for Sparse and Low-rank Optimization. National University of Singapore, June 2016. https://github.com/canyilu/LibADMM.

Canyi Lu, Jiashi Feng, Shuicheng Yan and Zhouchen Lin. A Unified Alternating Direction Method of Multipliers by Majorization Minimization. IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 40, pp. 527-541, 2018

The corresponding BiBTeX citations are given below:

```
@manual{lu2016libadmm,
    author       =   {Lu, Canyi},
    title        =   {A Library of {ADMM} for Sparse and Low-rank Optimization},
    organization =   {National University of Singapore},
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@article{lu2018unified,
    author       =   {Lu, Canyi and Feng, Jiashi and Yan, Shuicheng and Lin, Zhouchen},
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    year         =   {2018},
    volume       =   {40},
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```

TABLE 1: Applicability of the LibADMM package

Model	Problem		Function	Description and Reference
Sparse models	$\min_{\mathbf{x}} \ r(\mathbf{x})$ s.t. $\mathbf{A}\mathbf{x}=\mathbf{b}$	$r(\mathbf{x}) = \\|\mathbf{x}\\|_1$	l1	ℓ_1 [16]
		$r(\mathbf{x}) = \sum_{g\in\mathcal{G}} \\|\mathbf{x}_g\\|_2$	groupl1	Group Lasso [19]
		$r(\mathbf{x}) = \\|\mathbf{x}\\|_1 + \lambda_2\\|\mathbf{x}\\|_2^2$	elasticnet	Elastic net [21]
		$r(\mathbf{x}) = \\|\mathbf{x}\\|_1 + \lambda_2 \sum_{i=2}^{p} \|x_i - x_{i-1}\|$	fusedl1	Fused Lasso [17]
		$r(\mathbf{x}) = \\|\mathbf{A}\text{Diag}(\mathbf{x})\\|_*$	tracelasso	Trace Lasso [12]
		$r(\mathbf{x}) = \frac{1}{2}\\|\mathbf{x}\\|_{\text{ksp}}^2$	ksupport	k support norm [6]
	$\min_{\mathbf{x},\mathbf{e}} \ l(\mathbf{e}) + \lambda r(\mathbf{x})$ s.t. $\mathbf{A}\mathbf{x}+\mathbf{e}=\mathbf{b}$	$l(\mathbf{e}) = \\|\mathbf{e}\\|_1$ $l(\mathbf{e}) = \frac{1}{2}\\|\mathbf{e}\\|_2^2$	l1R	Reg. ℓ_1
			groupl1R	Reg. Group Lasso
			elasticnetR	Reg. Elastic net
			fusedl1R	Reg. Fused Lasso
			tracelassoR	Reg. Trace Lasso
			ksupportR	Reg. k support norm
Low-rank matrix models	$\min_{\mathbf{L},\mathbf{S}} \ \\|\mathbf{L}\\|_* + \lambda l(\mathbf{S}), \ \text{s.t.}\ \mathbf{X} = \mathbf{L} + \mathbf{S}$		rpca	Robust PCA [2]
	$\min_{\mathbf{X}} \ \\|\mathbf{X}\\|_*, \ \text{s.t.}\ \mathcal{P}_\Omega(\mathbf{X}) = \mathcal{P}_\Omega(\mathbf{M})$		lrmc	Low-rank matrix completion [1]
	$\min_{\mathbf{X},\mathbf{E}} \ \\|\mathbf{X}\\|_* + \lambda l(\mathbf{E}), \ \text{s.t.}\ \mathcal{P}_\Omega(\mathbf{X}) + \mathbf{E} = \mathbf{M}$		lrmcR	Reg. Low-rank matrix completion
	$\min_{\mathbf{X},\mathbf{E}} \ \\|\mathbf{X}\\|_* + \lambda l(\mathbf{E}), \ \text{s.t.}\ \mathbf{A} = \mathbf{B}\mathbf{X} + \mathbf{E}$		lrr	Low-rank representation [7]
	$\min_{\mathbf{Z},\mathbf{L},\mathbf{E}} \ \\|\mathbf{Z}\\|_* + \\|\mathbf{L}\\|_* + \lambda l(\mathbf{E})$ s.t. $\mathbf{X}\mathbf{Z} + \mathbf{L}\mathbf{X} - \mathbf{X} = \mathbf{E}$		latlrr	Latent low-rank representation [8]
	$\min_{\mathbf{X},\mathbf{E}} \ \\|\mathbf{X}\\|_* + \lambda_1\\|\mathbf{X}\\|_1 + \lambda_2 l(\mathbf{E})$ s.t. $\mathbf{A} = \mathbf{B}\mathbf{X} + \mathbf{E}$		lrsr	Low-rank and sparse representation [20]
	$\min_{\mathbf{L}_i,\mathbf{S}_i} \ \\|\mathbf{L}\\|_* + \lambda \sum_{i=1}^{m} \\|\mathbf{S}_i\\|_1,$ s.t. $\mathbf{X}_i = \mathbf{L} + \mathbf{S}_i, i = 1,\cdots,m, \mathbf{L} \geq 0, \mathbf{L}\mathbf{1} = \mathbf{1}$		rmsc	Robust multi-view spectral clustering [18]
	$\min_{\mathbf{Z}_i,\mathbf{E}_i} \ \sum_{i=1}^{K} (\\|\mathbf{Z}_i\\|_* + \lambda l(\mathbf{E}_i)) + \alpha\\|\mathbf{Z}\\|_{2,1}$ s.t. $\mathbf{X}_i = \mathbf{X}_i\mathbf{Z}_i + \mathbf{E}_i, i = 1,\cdots,K$		mlap	Multi-task low-rank affinity pursuit [4]
	$\min_{\mathbf{L},\mathbf{S}} \ \\|\mathbf{L}\\|_* + \lambda\\|\mathbf{C}\circ\mathbf{S}\\|_1, \ \text{s.t.}\ \mathbf{A} = \mathbf{L} + \mathbf{S}, 0 \leq \mathbf{L} \leq 1$		igc	Improved graph clustering [3]
	$\min_{\mathbf{P}} \ \langle \mathbf{P}, \mathbf{L} \rangle + \lambda\\|\mathbf{P}\\|_1, \ \text{s.t.}\ 0 \preceq \mathbf{P} \preceq \mathbf{I}, \text{Tr}(\mathbf{P}) = k$		sparsesc	Sparse spectral clustering [15]
Low-rank tensor models	$\min_{\boldsymbol{\mathcal{L}},\boldsymbol{\mathcal{S}}} \ \sum_{i=1}^{k} \alpha_i \\|\boldsymbol{\mathcal{L}}_{i(i)}\\|_* + \\|\boldsymbol{\mathcal{S}}\\|_1, \ \text{s.t.}\ \boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{S}}$		trpca_snn	Tensor robust PCA based on sum of nuclear norm [5]
	$\min_{\boldsymbol{\mathcal{X}}} \ \sum_{i=1}^{k} \alpha_i \\|\boldsymbol{\mathcal{X}}_{i(i)}\\|_*, \ \text{s.t.}\ \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) = \mathcal{P}_\Omega(\boldsymbol{\mathcal{M}})$		lrtc_snn	Low-rank tensor completion based on sum of nuclear norm [9]
	$\min_{\boldsymbol{\mathcal{X}},\boldsymbol{\mathcal{E}}} \ \sum_{i=1}^{k} \alpha_i \\|\boldsymbol{\mathcal{X}}_{i(i)}\\|_* + \lambda l(\boldsymbol{\mathcal{E}})$ s.t. $\mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}}$		lrtcR_snn	Reg. low-tank tensor completion based on sum of nuclear norm
	$\min_{\boldsymbol{\mathcal{L}},\boldsymbol{\mathcal{S}}} \ \\|\boldsymbol{\mathcal{L}}\\|_* + \lambda\\|\boldsymbol{\mathcal{S}}\\|_1, \ \text{s.t.}\ \boldsymbol{\mathcal{X}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{S}}$		trpca_tnn	Tensor Robust PCA based on tensor nuclear norm [11]
	$\min_{\boldsymbol{\mathcal{X}}} \ \\|\boldsymbol{\mathcal{X}}\\|_*, \ \text{s.t.}\ \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) = \mathcal{P}_\Omega(\boldsymbol{\mathcal{M}})$		lrtc_tnn	Low-rank tensor completion based on tensor nuclear norm [13]
	$\min_{\boldsymbol{\mathcal{X}},\boldsymbol{\mathcal{E}}} \ \\|\boldsymbol{\mathcal{X}}\\|_* + \lambda l(\boldsymbol{\mathcal{E}}), \ \text{s.t.}\ \mathcal{P}_\Omega(\boldsymbol{\mathcal{X}}) + \boldsymbol{\mathcal{E}} = \boldsymbol{\mathcal{M}}$		lrtcR_tnn	Reg. low-rank tensor completion based on tensor nuclear norm [13]
	$\min_{\boldsymbol{\mathcal{X}}} \ \\|\boldsymbol{\mathcal{X}}\\|_*, \ \text{s.t.}\ \mathbf{y} = \Phi(\boldsymbol{\mathcal{X}})$		lrtr_Gaussian_tnn	Low-rank tensor recovery from Gaussian measurements based on tensor nuclear norm [13]

REFERENCES

[1] E. Candès and B. Recht. Exact matrix completion via convex optimization. *Foundations of Computational mathematics*, 9(6):717–772, 2009.
[2] E. J. Candès, X. D. Li, Y. Ma, and J. Wright. Robust principal component analysis? *Journal of the ACM*, 58(3), 2011.
[3] Y. Chen, S. Sanghavi, and H. Xu. Improved graph clustering. *IEEE Transactions on Information Theory*, 60(10):6440–6455, 2014.
[4] B. Cheng, G. Liu, J. Wang, Z. Huang, and S. Yan. Multi-task low-rank affinity pursuit for image segmentation. In *ICCV*, pages 2439–2446. IEEE, 2011.
[5] D. Goldfarb and Z. Qin. Robust low-rank tensor recovery: Models and algorithms. *SIAM Journal on Matrix Analysis and Applications*, 35(1):225–253, 2014.
[6] H. Lai, Y. Pan, C. Lu, Y. Tang, and S. Yan. Efficient k-support matrix pursuit. In *ECCV*, 2014.
[7] G. Liu, Z. Lin, S. Yan, J. Sun, Y. Yu, and Y. Ma. Robust recovery of subspace structures by low-rank representation. *TPAMI*, 2013.
[8] G. Liu and S. Yan. Latent low-rank representation for subspace segmentation and feature extraction. In *ICCV*, 2011.
[9] J. Liu, P. Musialski, P. Wonka, and J. Ye. Tensor completion for estimating missing values in visual data. *TPAMI*, 35(1):208–220, 2013.
[10] C. Lu. *A Library of ADMM for Sparse and Low-rank Optimization*. National University of Singapore, June 2016. https://github.com/canyilu/LibADMM.

[11] C. Lu, J. Feng, Y. Chen, W. Liu, Z. Lin, and S. Y. Yan. Tensor robust principal component analysis: Exact recovery of corrupted low-rank tensors via convex optimization. In *CVPR*. IEEE, 2016.

[12] C. Lu, J. Feng, Z. Lin, and S. Yan. Correlation adaptive subspace segmentation by trace Lasso. In *ICCV*, 2013.

[13] C. Lu, J. Feng, Z. Lin, and S. Yan. Exact low tubal rank tensor recovery from gaussian measurements. In *International Joint Conference on Artificial Intelligence*, 2018.

[14] C. Lu, J. Feng, S. Yan, and Z. Lin. A unified alternating direction method of multipliers by majorization minimization. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 40(3):527–541, 2018.

[15] C. Lu, S. Yan, and Z. Lin. Convex sparse spectral clustering: Single-view to multi-view. *TIP*, 25(6):2833–2843, 2016.

[16] R. Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society. Series B (Methodological)*, pages 267–288, 1996.

[17] R. Tibshirani, M. Saunders, S. Rosset, J. Zhu, and K. Knight. Sparsity and smoothness via the fused lasso. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 67(1):91–108, 2005.

[18] R. Xia, Y. Pan, L. Du, and J. Yin. Robust multi-view spectral clustering via low-rank and sparse decomposition. In *AAAI*, pages 2149–2155, 2014.

[19] M. Yuan and Y. Lin. Model selection and estimation in regression with grouped variables. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 68(1):49–67, 2006.

[20] L. Zhuang, H. Gao, Z. Lin, Y. Ma, X. Zhang, and N. Yu. Non-negative low rank and sparse graph for semi-supervised learning. In *CVPR*, pages 2328–2335. IEEE, 2012.

[21] H. Zou and T. Hastie. Regularization and variable selection via the elastic net. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 67(2):301–320, 2005.